1 Geometry and Linear Algebra

1.1 Geometry

Pole and polars. Diameters and conjugate diameters. Systems of conics. Polar equation of a conic referred to a focus as pole. Equations of tangent, normal, chord of contact. Sphere: General Equation. Great circle, Sphere through the intersection of two spheres. Radical Plane, Tangent, Normal. Cone: Right circular cone. General homogeneous second degree equation. Section of cone by a plane as a conic and as a pair of lines. Condition for three perpendicular generators. Reciprocal cone. Cylinder: Generators parallel to either of the axes, general form of equation. Right-circular cylinder. Ellipsoid, Hyperboloid, Paraboloid: Canonical equations only. Tangent planes, Normal, Enveloping cone. Generating lines of hyperboloid of one sheet and hyperbolic paraboloid.

1.2 Vector spaces

Vectors in $\mathbb{R}^n$, notions of linear independence and dependence, linear span of a set of vectors, vector subspaces of $\mathbb{R}^n$, basis of a vector subspace. Systems of linear equations, matrices and Gauss elimination, row space, null space, column space, rank of a matrix. Vector spaces (over $\mathbb{R}$ or $\mathbb{C}$), subspaces, algebra of subspaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension.

1.3 Linear transformations

Linear transformations, null space, range, rank and nullity of a linear transformation, matrix representation of a linear transformation, Different notion of matrices, Eigen values, Eigen vectors and characteristic equation of a matrix. Cayley-Hamilton theorem. Algebra of linear transformations. Isomorphisms. Isomorphism theorems, invertibility and isomorphisms, change of coordinate matrix.

2 Algebra

2.1 Classical algebra

Theory of equations: Relation between roots and coefficients, transformation of equation, Descartes rule of signs, cubic and biquadratic equation. Inequalities, weighted A.M.-G.M.-H.M. inequality, Cauchy-Schwarz inequality.

2.2 Groups

Definition and examples of groups including permutation groups, dihedral groups and quaternion groups. Elementary properties of groups. Subgroups and examples of subgroups, centraliser, normaliser, center of a group, product of two subgroups.

2.3 Cyclic groups and applications to number theory

Properties of cyclic groups, classification of subgroups of cyclic groups. Cycle notation for permutations, properties of permutations, even and odd permutations, alternating group, properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem.

2.4 Homomorphisms, isomorphisms and Cayley's theorem

External direct product of a finite number of groups, normal subgroups, quotient groups, Cauchy's theorem for finite abelian groups. Group homomorphisms, properties of homomorphisms, properties of isomorphisms. First isomorphism theorem, Cayley's theorem.

2.5 Rings

Definition and examples of rings, properties of rings, subrings, integral domains and fields, characteristic of a ring.