1 Real Analysis

1.1 Point-set topology of $\mathbb{R}$

Review of algebraic and order properties of $\mathbb{R}$, ϵ-neighborhood of a point in $\mathbb{R}$. Idea of countable sets, uncountable sets and uncountability of $\mathbb{R}$. Bounded above sets, bounded below sets, bounded sets, unbounded sets. Suprema and infima. Construction of $\mathbb{R}$ from $\mathbb{Q}$, Cantor's nested interval Theorem, Completeness property of $\mathbb{R}$ and its equivalent properties. The Archimedean property, density of rational (and irrational) numbers in $\mathbb{R}$, intervals. Limit points of a set, isolated points, open set, closed set, derived set, Bolzano-Weierstrass theorem for sets, compact sets in $\mathbb{R}$, Heine-Borel Theorem.

1.2 Convergence of sequences in $\mathbb{R}$

Sequences, bounded sequence, convergent sequence, limit of a sequence, lim inf, lim sup. Limit theorems. Monotone sequences, monotone convergence theorem. Subsequences, divergence criteria. Monotone subsequence theorem, Bolzano-Weierstrass theorem for sequences. Cauchy sequence, Cauchy's convergence criterion.

1.3 Convergence of series in $\mathbb{R}$

Infinite series, convergence and divergence of infinite series, Cauchy criterion, tests for convergence: comparison test, limit comparison test, D'Alembert's test, Raabe's test, Cauchy's nth root test, Gauss test, Logarithmic test, Integral test. Alternating series, Leibniz test. Absolute and conditional convergence, Rearrangement of series, Riemann's theorem on conditionally convergent series.

2 Theory of Real Functions

2.1 Continuity in $\mathbb{R}$

Limits of functions (ϵ-δ approach), sequential criterion for limits, divergence criteria. Limit theorems, one sided limits. Infinite limits and limits at infinity. Continuous functions, sequential criterion for continuity and discontinuity. Algebra of continuous functions. Continuous functions on closed and bounded interval, intermediate value theorem, location of roots theorem, preservation of intervals theorem. Classification of discontinuity, discontinuity of monotonic functions. Uniform continuity, non-uniform continuity criteria, uniform continuity theorem on compact sets.

2.2 Differentiability in $\mathbb{R}$

Differentiability of a function at a point and in an interval, Caratheodory's theorem, algebra of differentiable functions. Relative extrema, interior extremum theorem. Rolle's theorem. Mean value theorem: Lagrange's mean value theorem, Cauchy's mean value theorem, Darboux's theorem on derivatives. Applications of mean value theorem to inequalities and approximation of polynomials.

2.3 Taylor's theorem and applications

Taylor's theorem with Lagrange's form of remainder, Taylor's theorem with Cauchy's form of remainder and Young's form of remainder, application of Taylor's theorem to convex functions, Jensen's inequality, relative extrema. Taylor's series and Maclaurin's series expansions of exponential and trigonometric functions, $\log(1 + x)$, $1/(ax + b)$ and $(x + 1)^n$. Application of Taylor's theorem to inequalities. L'Hospital's rule.

2.4 Advanced calculus

Higher order derivatives, Leibnitz rule, concavity and inflection points, envelopes, asymptotes, curvature, curve tracing in cartesian coordinates. Reduction formulae, derivations and illustrations of reduction formulae, parametric equations, parameterizing a curve, arc length of a curve, arc length of parametric curves, area under a curve, area and volume of surface of revolution.