1 Algebraic Number Theory

References:

1. *A Brief Guide to Algebraic Number Theory* by Peter Swinnerton-Dyer.
2. *Algebraic Theory of Numbers* by Pierre Samuel.
3. *Number Fields* by Daniel Marcus.
4. *Algebraic Number Theory* by Jürgen Neukirch.
5. *Local Fields* by Jean-Pierre Serre.

Week 1: Study of number fields, definition of the ring of integers. Definition of norm and trace.

Week 2: Definition of absolute and relative discriminant. Computation of discriminant. Computation of the ring of integers.

Week 3: Definition and properties of Dedekind domains. Proof that the ring of integers is a Dedekind domain. Factorisation of extension of prime ideals in a finite extension of number fields.

Week 4: Embeddings of a number field in complex numbers. A result from geometry of numbers. Finiteness of class groups.

Week 5: Computation of class groups, including several examples. Applications to Diophantine equations of computations of class groups.

Week 6: Dirichlet's unit theorem.

Week 7: Extension and norm of ideals in field extensions. Maps between class groups of extensions. Decomposition subgroups, inertia subgroups, Frobenius elements etc. Localisation, residue field.

Week 8: Valuations in a number fields. Local fields. Hensel's lemma and applications.

Week 9: Field extensions of local fields, ramification, different, inertia subgroups etc.

Week 10: Study of special number fields. Imaginary quadratic fields, real quadratic fields, cubic fields, cyclotomic fields.

Week 11: Definition of ray class field as a generalisation of ideal class group. Some statements from class field theory without proofs.

Week 12: Definition of zeta functions and L-functions. Statements of their analytic properties without proofs. Dirichlet Class number formula.