# 1 Group Theory

## 1.1 Isomorphism theorems and automorphisms

Isomorphism theorems, correspondence theorem, automorphism of groups, inner automorphism, automorphism groups of some familiar groups. Direct product of groups, internal direct products, semidirect products.

## 1.2 Cyclic groups

Characterisation of cyclic groups, finitely generated abelian groups and structure theorem.

## 1.3 Group actions and Sylow theorems

Group actions and permutation representations, groups acting on themselves by left multiplication, Cayley's theorem and extended Cayley's theorem, $p$-group, Cauchy's theorem (McKay's approach), finite $p$-group. Groups acting on themselves by conjugation, the class equation, some applications (To prove that the center of a $p$-group is nontrivial). The Sylow theorems, Classification of finite groups of order $pq, p^2q, p^2q^2$ ($p, q$ are primes) etc. Simple groups, simplicity of $A_n$; Simplicity of groups of order $\leq 60$.

## 1.4 Solvable and nilpotent groups

$p$-groups, nilpotent groups, and solvable Groups.