# 1 Multivariate Calculus

## 1.1 Continuity and differentiability for $f : \mathbb{R}^n \to \mathbb{R}$

Limit, continuity and differentiability of functions from $\mathbb{R}^n$ to $\mathbb{R}$ (with special emphasis from $\mathbb{R}^2$ to $\mathbb{R}$), partial derivatives, directional derivatives, the gradient, maximal and normal property of the gradient, tangent planes, extrema of function, method of Lagrange multipliers, constrained optimisation problems.

## 1.2 Continuity and differentiability for $f : \mathbb{R}^n \to \mathbb{R}^m$

Limit, continuity and differentiability of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, total derivative as a linear map, Jacobian matrix, chain rule, Mean value theorem, sufficient condition for differentiability, higher order derivatives, Taylor's formula for functions from $\mathbb{R}^n$ to $\mathbb{R}$, inverse function theorem and implicit function theorem.

## 1.3 Curves in $\mathbb{R}^n$ and line integrals

Curves in $\mathbb{R}^n$; Integration of vector functions. Line integrals: definition of vector field, divergence and curl. Applications of line integrals: Fundamental theorem for line integrals, conservative vector fields, independence of path; The Frenet-Serret Equations; Geometry of Curves in $\mathbb{R}^3$.

## 1.4 Multidimensional integration and Stokes' theorem

Double integration: over rectangular and non-rectangular regions, double integrals in polar co-ordinates; Green's theorem; Parameterised Surfaces in $\mathbb{R}^3$; Surface area; Surface integrals; Integrals over parametrically defined surfaces; Stokes' theorem; Triple integrals: triple integral over a parallelepiped and solid regions. Volume by triple integrals: cylindrical and spherical co-ordinates; Change of variables in double integrals and triple integrals. The divergence theorem.