General Mechanics

Semester 7, 2026

Based on lectures by Anup Bandyopadhyay
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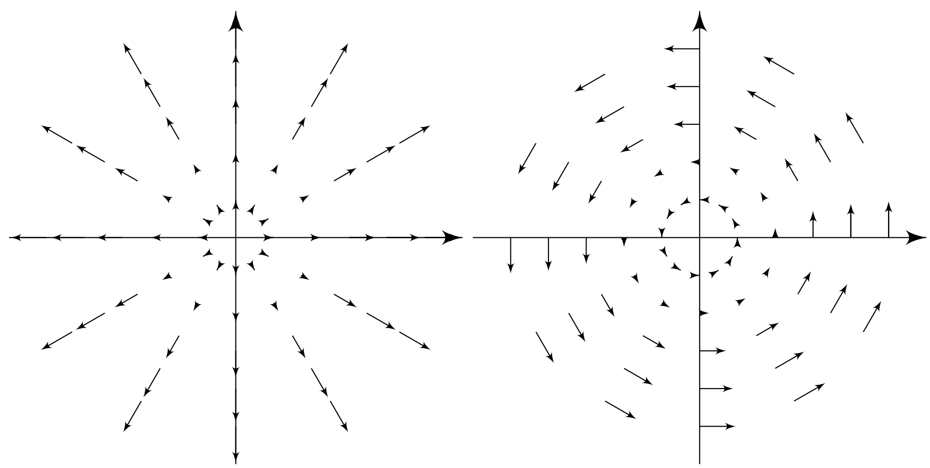

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List of Lectures

1. Lagrangian Mechanics

1.1 The Lagrangian

Lecture 1
1^{st} Aug, 2026

Definition 1: Lagrangian

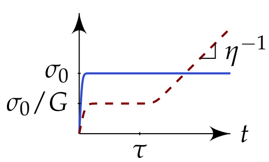
The **Lagrangian** is defined as $L = T - V$ satisfying

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0. \tag{1.1}$$

1.2 D'Alembert's principle

Lecture 2
5^{th} Aug, 2026

The work done by the applied forces along with the work done by the reversed effective forces for any virtual displacements consistent with the conditions for the bilateral constraints is zero; in other words, if a material system is under the action of a system of externally applied forces and subjected to bilateral constraints only, then the reversed effective force and the externally applied body force, acting on each particle of the material system, form a system of forces in equilibrium



$$\sum_i \left(\vec{F}_i - m_i \dot{\vec{v}} - \dot{m}_i \vec{v}_i\right) \cdot \delta \vec{r}_i = 0$$

2. Hamiltonian Mechanics

2.1 Symmetries and Noether's theorem

Given

$$F[x] = \int_\alpha^\beta f(x, \dot{x}, t)\, \mathrm{d}t,$$

suppose we change variables by the transformation $t \mapsto t^*(t)$ and $x \mapsto x^*(t^*)$. Then we have a new independent variable and a new function. This gives

$$F[x] \mapsto F^*[x^*] = \int_{\alpha^*}^{\beta^*} f(x^*, \dot{x}^*, t^*)\, \mathrm{d}t^*$$

with $\alpha^* = t^*(\alpha)$ and $\beta^* = t^*(\beta)$. So there are some transformations that are particularly interesting

Definition 2

If $F^*[x^*] = F[x]$ for all x, α and β, then the transformation $*$ is a **symmetry**.

Theorem 1: Noether's theorem

For every continuous symmetry of $F[x]$, the solutions (i.e. the stationary points of $F[x]$) will have a corresponding conserved quantity.

Proof. The proof is trivial and left as an exercise to the reader. ∎

Corollary 1

Corollary to Noether's theorem.

Example 1

An example of a symmetry.

Counterexample 1

A non-example of a symmetry.

Observation. Observe the following phenomena.

Question 1. Compute the follow integral over $\mathbb{C}$

$$\int_{-1}^{1} \frac{1}{x}\sqrt{\frac{1+x}{1-x}} \log\left(\frac{2x^2+2x+1}{2x^2-2x+1}\right).$$

Solution. The answer is

$$\int_{-1}^{1} \frac{1}{x}\sqrt{\frac{1+x}{1-x}} \log\left(\frac{2x^2+2x+1}{2x^2-2x+1}\right) = 4\pi \operatorname{arccot}\sqrt{\phi}.$$

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